Question1

If $\frac{1}{6}\sin\theta, \cos\theta, \tan\theta$ are in G.P., then the general solution of θ is MHT CET 2025 (22 Apr Shift 2)

Options:

A. $2n\pi \pm \frac{\pi}{3}, n \in \mathbb{Z}$

B. $\mathrm{n}\pi + \frac{\pi}{3}, \mathrm{n} \in \mathbb{Z}$

C. $\mathrm{n}\pi + \frac{\pi}{4}, \mathrm{n} \in \mathbb{Z}$

D. $2\mathrm{n}\pi \pm \frac{\pi}{6}, \mathrm{n} \in \mathbb{Z}$

Answer: A

Solution:

Let the three terms be in G.P.:

$$\frac{1}{6}\sin\theta,\ \cos\theta,\ \tan\theta$$

For a geometric progression, the middle squared equals the product of the extremes:

$$\cos^2\theta = \frac{1}{6}\sin\theta\cdot\tan\theta = \frac{1}{6}\sin\theta\cdot\frac{\sin\theta}{\cos\theta} = \frac{1}{6}\frac{\sin^2\theta}{\cos\theta}.$$

Hence

$$6\cos^3\theta = \sin^2\theta.$$

Divide by $\cos^2\theta$ (note $\cos\theta \neq 0$ since $\tan\theta$ must be defined):

$$6\cos\theta = \tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta} = \frac{1-\cos^2\theta}{\cos^2\theta}.$$

Let $t = \cos\theta$. Then

$$6t = \frac{1-t^2}{t^2} \Rightarrow 6t^3 + t^2 - 1 = 0 \Rightarrow (2t-1)(3t^2+2t+1) = 0.$$

Only the real root is $t = \frac{1}{2}$, i.e. $\cos\theta = \frac{1}{2}$.

Therefore,

$$\theta = 2\pi n \pm \frac{\pi}{3}, \quad n \in \mathbb{Z}.$$

So the general solution is $2n\pi \pm \pi/3$, $n \in Z$.

Question2

The money invested in a company is compounded continuously. ₹400 invested today becomes ₹800 in 6 years, then at the end of 33 years, it will become _$(\sqrt{2}=1\cdot 4142)$ MHT CET 2025 (20 Apr Shift 1)

Options:

A. $9050 \cdot 88$

B. $18101 \cdot 76$

C. $6788 \cdot 16$

D. $12067 \cdot 84$

Answer: B

Solution:

Given: ₹400 → ₹800 in 6 yrs, continuous compounding.

$$800 = 400e^{6r} \implies e^{6r} = 2 \implies r = \frac{\ln 2}{6}.$$

After 33 yrs:

$$A = 400e^{33r} = 400e^{\frac{33}{6}\ln 2} = 400 \cdot 2^{5.5} = 400 \cdot 32\sqrt{2} \approx 18101.76.$$

✅ Answer: **₹18101.76 (Option B)**

Question3

If x, y, z are in Arithmetic Progression and $\tan^{-1} x, \tan^{-1} y, \tan^{-1} z$ are also in Arithmetic progression, where $x, z > 0$ and $xz < 1, y < 1$, then MHT CET 2024 (10 May Shift 2)

Options:

A. $x = y = z$

B. $2x = 3y = 6z$

C. $6x = 3y = 2z$

D. $6x = 4y = 3z$

Answer: A

Solution:

Given, x, y, z are in A.P.

$\therefore \quad 2y = x + z \ldots (i)$
Also, $\tan^{-1} x, \tan^{-1} y, \tan^{-1} z$ are in A.P.

$$\therefore \quad 2\tan^{-1} y = \tan^{-1} x + \tan^{-1} z$$
$$\Rightarrow \tan^{-1}\left(\frac{2y}{1-y^2}\right) = \tan^{-1}\left(\frac{x+z}{1-xz}\right)$$
$$\Rightarrow \frac{2y}{1-y^2} = \frac{x+z}{1-xz}$$
$$\Rightarrow \frac{2y}{1-y^2} = \frac{2y}{1-xz} \ldots [From(i)]$$
$$\Rightarrow 1 - y^2 = 1 - xz$$
$$\Rightarrow y^2 = xz$$

$\therefore \quad x, y, \mathrm{z}$ are in G.P...(ii)

From (i) and (ii), we get

$x = y = \mathbf{z}$

$x = y = \mathrm{z}$

Question4

If $\sin(\theta^\circ - \alpha), \sin\theta$ and $\sin(\theta + \alpha)$ are in H.P., then the value of $\cos^2\theta$ is MHT CET 2024 (03 May Shift 2)

Options:

A. $1 - 2\cos^2\frac{\alpha}{2}$

B. $1 + 2\cos^2\frac{\alpha}{2}$

C. $1 - 4\cos^2\frac{\alpha}{2}$

D. $1 + 4\cos^2\frac{\alpha}{2}$

Answer: A

Solution:

$\sin(\theta-\alpha), \sin\theta$ and $\sin(\theta+\alpha)$ are in H.P.

$\Rightarrow \frac{1}{\sin(\theta-\alpha)}, \frac{1}{\sin\theta}, \frac{1}{\sin(\theta+\alpha)}$ will be in A.P.

$$\therefore \quad \frac{2}{\sin\theta} = \frac{1}{\sin(\theta-\alpha)} + \frac{1}{\sin(\theta+\alpha)}$$

$$\Rightarrow \frac{2}{\sin\theta} = \frac{\sin(\theta+\alpha)+\sin(\theta-\alpha)}{\sin(\theta-\alpha)\sin(\theta+\alpha)}$$

$$\Rightarrow \frac{2}{\sin\theta} = \frac{2\sin\theta\cos\alpha}{\sin^2\theta-\sin^2\alpha}$$

$$\Rightarrow \sin^2\theta-\sin^2\alpha = \sin^2\theta\cos\alpha$$

$$\Rightarrow \sin^2\theta(1-\cos\alpha) = \sin^2\alpha$$

$$\Rightarrow \sin^2\theta\left(2\sin^2\frac{\alpha}{2}\right) = 4\sin^2\frac{\alpha}{2}\cos^2\frac{\alpha}{2}$$

$$\Rightarrow 1-\cos^2\theta = 2\cos^2\frac{\alpha}{2}$$

$$\Rightarrow \cos^2\theta = 1-2\cos^2\frac{\alpha}{2}$$

Question5

A radioactive substance, with initial mass m_0, has a half-life of h days. Then its initial decay rate is given by MHT CET 2023 (13 May Shift 2)

Options:

A. $\frac{\mathrm{m}_0}{\mathrm{h}}\log 2$

B. $\mathrm{m}_0\ \mathrm{h}\log 2$

C. $-\frac{\mathrm{m}_0}{\mathrm{h}}\log 2$

D. $-m_0\ \mathrm{h}\log 2$

Answer: C

Solution:

Let m be the mass of substance at time t. Then,

$$\frac{dm}{dt} = -km, \text{ where } k > 0$$
$$\Rightarrow \frac{dm}{m} = -kdt$$

Integrating on both sides, we get

$$\log m = -kt + c$$

When $t = 0,\ m = m_0$

$$\therefore \quad \log m_0 = 0 + c$$

$$\Rightarrow c = \log m_0$$

$$\therefore \quad \log m = -kt + \log m_0$$

$$\Rightarrow \log \frac{m}{m_0} = -kt$$

When $t = h, m = \frac{1}{2} m_0$

$$\therefore \quad \log\left(\frac{\frac{1}{2}m_0}{m_0}\right) = -kh$$

$$\Rightarrow \log \frac{1}{2} = -kh$$
$$\Rightarrow \log 2 = kh$$

$\Rightarrow k = \frac{\log 2}{h}$ ...(i)

Initial decay rate,

$$\frac{dm}{dt} = -km_0$$

$= \frac{-m_0}{h} \log 2$ ...[From (i)]

Question6

The sides of a triangle are three consecutive natural numbers and its largest angle is twice the smallest one, then the sides of the triangle (in units) are MHT CET 2023 (13 May Shift 1)

Options:

A. $3, 4, 5$

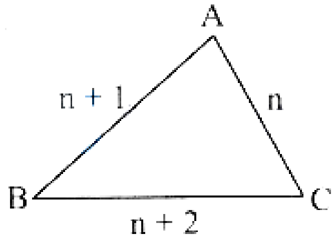
B. $4, 5, 6$

C. $5, 6, 7$

D. $2, 3, 4$

Answer: B

Solution:



Let $\mathrm{AC} = \mathrm{n}, \mathrm{AB} = \mathrm{n} + 1, \mathrm{BC} = \mathrm{n} + 2 \therefore$ Largest angle is A and smallest angle is B.

$$\therefore \quad \mathrm{A} = 2\,\mathrm{B}$$

Since $\mathrm{A} + \mathrm{B} + \mathrm{C} = 180^\circ \therefore \quad 3\,\mathrm{B} + \mathrm{C} = 180^\circ$

$$\Rightarrow \mathrm{C} = 180^\circ - 3\,\mathrm{B}$$
$$\Rightarrow \sin \mathrm{C} = \sin(180^\circ - 3\,\mathrm{B}) = \sin 3\,\mathrm{B}$$

By sine rule,

$$\frac{\sin A}{n+2}=\frac{\sin B}{n}=\frac{\sin C}{n+1}$$

$$\Rightarrow \frac{\sin 2B}{n+2}=\frac{\sin B}{n}=\frac{\sin 3B}{n+1}$$

$$\Rightarrow \frac{2\sin B\cos B}{n+2}=\frac{\sin B}{n}=\frac{3\sin B-4\sin^3 B}{n+1}$$

$$\Rightarrow \frac{2\cos B}{n+2}=\frac{1}{n}=\frac{3-4\sin^2 B}{n+1}$$

$$\therefore \cos B=\frac{n+2}{2n}, 3-4\sin^2 B=\frac{n+1}{n}$$

$$\therefore \quad 3-4\left(1-\cos^2 B\right)=\frac{n+1}{n}$$

$$\therefore \quad -4+4\left(\frac{n+2}{2n}\right)^2=\frac{n+1}{n}$$

$$\Rightarrow -1+\frac{n^2+4n+4}{n^2}=\frac{n+1}{n}$$

$$\Rightarrow -n^2+n^2+4n+4=n^2+n$$

$$\Rightarrow n^2-3n-4=0$$

$$\Rightarrow (n+1)(n-4)=0$$

$$\Rightarrow n=-1 \text{ or } n=4$$

But n cannot be negative.

$$\therefore \mathrm{n}=4$$

$\therefore$ The sides of the Δ are 4, 5, 6.

Question7

x, y, z are in G.P. and $\tan^{-1} x, \tan^{-1} y, \tan^{-1} z$ are in A.P., then MHT CET 2023 (13 May Shift 1)

Options:

A. $6x = 4y = 3z$

B. $2x = 3y = 6z$

C. $6x = 3y = 2z$

D. $x = y = z$

Answer: D

Solution:

x, y, z are in G.P.

$$\Rightarrow y^2 = x\text{z}$$

Also, $\tan^{-1} x, \tan^{-1} y, \tan^{-1} z$ are in A.P.

$$\Rightarrow 2\tan^{-1} y = \tan^{-1} x + \tan^{-1} \text{z}$$
$$\Rightarrow \tan^{-1}\left(\frac{2y}{1-y^2}\right) = \tan^{-1}\left(\frac{x+\text{z}}{1-x\text{z}}\right)$$
$$\Rightarrow \frac{2y}{1-y^2} = \frac{x+\text{z}}{1-x\text{z}}$$
$$\Rightarrow \frac{2y}{1-x\text{z}} = \frac{x+\text{z}}{1-x\text{z}}$$

$$\Rightarrow 2y = x + \text{z}$$

$\Rightarrow x, y$, z are in A.P.From (i) and (ii), we get

$$x = y = \text{Z}$$

Question8

Angles of a triangle are in the ratio $4:1:1$. Then the ratio of its greatest side to its perimeter is MHT CET 2023 (13 May Shift 1)

Options:

A. $3:(2+\sqrt{3})$

B. $\sqrt{3}:(2+\sqrt{3})$

C. $\sqrt{3}:(2-\sqrt{3})$

D. $1:(2+\sqrt{3})$

Answer: B

Solution:

Let the angles of the triangle be $4x$, x and x.

$$\therefore 4x+x+x=180^\circ \Rightarrow 6x=180^\circ \Rightarrow x=30^\circ$$

By sine rule,

$$\begin{aligned} &\frac{\sin 120^\circ}{\mathrm{a}}=\frac{\sin 30^\circ}{\mathrm{b}}=\frac{\sin 30^\circ}{\mathrm{c}} \\ \therefore\quad &\mathrm{a}:(\mathrm{a}+\mathrm{b}+\mathrm{c}) \\ &=(\sin 120^\circ):(\sin 120^\circ+\sin 30^\circ+\sin 30^\circ) \\ &=\frac{\sqrt{3}}{2}:\frac{\sqrt{3}+2}{2}=\sqrt{3}:\sqrt{3}+2 \end{aligned}$$

Question9

Rate of increase of bacteria in a culture is proportional to the number of bacteria present at that instant and it is found that the number doubles in 6 hours. The number of bacteria becomes times at the end of 18 hours. MHT CET 2023 (13 May Shift 1)

Options:

A. 9

B. 6

C. 8

D. 3

Answer: C

Solution:

Let P_0 be the initial population and let the population after t years be P. Then,

$$\frac{\mathrm{dP}}{\mathrm{dt}} = \mathrm{kP}, \text{ where } \mathrm{k} > 0$$
$$\Rightarrow \frac{\mathrm{dP}}{\mathrm{P}} = \mathrm{kdt}$$

Integrating on both sides, we get

$$\log \mathrm{P} = \mathrm{kt} + \mathrm{c}$$
$$\text{When } \mathrm{t} = 0, \mathrm{P} = \mathrm{P}_0$$
$$\therefore \quad \log \mathrm{P}_0 = 0 + \mathrm{c}$$
$$\Rightarrow c = \log \mathrm{P}_0$$
$$\log \mathrm{P} = \mathrm{kt} + \log \mathrm{P}_0$$
$$\Rightarrow \log \frac{\mathrm{P}}{\mathrm{P}_0} = \mathrm{kt}$$
$$\therefore \quad \log \mathrm{P} = \mathrm{kt} + \log \mathrm{P}_0$$
$$\Rightarrow \log \frac{\mathrm{P}}{\mathrm{P}_0} = \mathrm{kt}$$
$$\text{When } \mathrm{t} = 6 \text{ hrs}, \mathrm{P} = 2\mathrm{P}_0$$
$$\therefore \quad \log \frac{2\mathrm{P}_0}{\mathrm{P}_0} = 6\mathrm{k}$$
$$\Rightarrow \mathrm{k} = \frac{\log 2}{6}$$

$$\therefore \quad \log \frac{P}{P_0} = \frac{\log 2}{6} t$$

When t $= 18$hrs, we have

$$\log \frac{P}{P_0} = \frac{\log 2}{6} \times 18$$

$$= 3 \log 2$$

$$\therefore \quad \log \frac{P}{P_0} = \log 8$$

$$\Rightarrow \mathrm{P} = 8\mathrm{P}_0$$

Question10

The lengths of sides of a triangle are three consecutive natural numbers and its largest angle is twice the smallest one. Then the length of the sides of the triangle (in units) are MHT CET 2023 (12 May Shift 2)

Options:

A. $3, 4, 5$

B. $4, 5, 6$

C. $5, 6, 7$

D. $2, 3, 4$

Answer: B

Solution:

Let $a, a+1, a+2$ be the sides of the triangle and $\mathrm{A}, \mathrm{B}, \mathrm{C}$ be the angles opposite to them respectively.According to the given condition,

$$C = 2A$$

$$\therefore \quad \sin C = \sin 2A$$

$$\sin C = 2 \sin A \cos A$$

Note that $\frac{\sin A}{a} = \frac{\sin C}{a+2} = k$

$$\Rightarrow \sin A = ka \text{ and } \sin C = k(a+2)$$

$$\text{Also, } \cos A = \frac{(a+1)^2 + (a+2)^2 - a^2}{2(a+1)(a+2)}$$

$$= \frac{a^2+2a+1+a^2+4a+4-a^2}{2(a^2+3a+2)}$$

$$= \frac{a^2+6a+5}{2(a^2+3a+2)}$$

$$\therefore \quad (i) \Rightarrow k(a+2) = 2 \times ka \times \frac{a^2+6a+5}{2(a^2+3a+2)}$$

$$\therefore \quad a+2 = \frac{a\left(a^2+6a+5\right)}{(a^2+3a+2)}$$

$$\therefore \quad (a+2)\left(a^2+3a+2\right) = a^3 + 6a^2 + 5a$$

$$\therefore \quad a^3 + 5a^2 + 8a + 4 = a^3 + 6a^2 + 5a$$

$$\therefore \quad a^2 - 3a - 4 = 0$$

$$\therefore \quad (a-4)(a+1) = 0$$

$$\Rightarrow a = 4 \text{ or } -1$$

But a $= -1$ is not possible.$\therefore$ 4, 5, 6 are the lengths of the sides of the triangle.

Question11

If x, y, z are in A.P. and $\tan^{-1} x, \tan^{-1} y$ and $\tan^{-1} z$ are also in A.P., then
MHT CET 2023 (10 May Shift 2)

Options:

A. $x = y =$ z

B. $2x = 3y = 6z$

C. $6x = 3y = 2z$

D. $6x = 4y = 3z$

Answer: A

Solution:

Given, x, y, z are in A.P.$\therefore \quad 2y = x + z \ldots (i)$Also,

$\tan^{-1} x, \tan^{-1} y, \tan^{-1} z$ are in A.P.

$$\therefore \quad 2\tan^{-1} y = \tan^{-1} x + \tan^{-1} z$$

$$\Rightarrow \tan^{-1}\left(\frac{2y}{1-y^2}\right) = \tan^{-1}\left(\frac{x+z}{1-xz}\right)$$

$$\Rightarrow \frac{2y}{1-y^2} = \frac{x+z}{1-xz}$$

$$\Rightarrow \frac{2y}{1-y^2} = \frac{2y}{1-xz} \ldots [From(i)]$$

$$\Rightarrow 1 - y^2 = 1 - xz$$

$$\Rightarrow y^2 = xz$$

$\therefore \quad x, y$, z are in G.P. $\ldots (ii)$From (i) and (ii), we get $x = y =$ z

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Question12

The sum to 10 terms of the series $1 \times 3^2 + 2 \times 5^2 + 3 \times 7^2 + \ldots\ldots\ldots\ldots\ldots$ is MHT CET 2020 (20 Oct Shift 1)

Options:

A. 13,495

B. ·15, 595

C. 13, 000

D. 13,695

Answer: D

Solution:

$1 \times 3^2 + 2 \times 5^2 + 3 \times 7^2 +$

Term : $\rightarrow 2(2q+1)^2$

$S = \sum_{\varepsilon=1}^{10} z(2q+1)^2$

$S = \sum_{\varepsilon=1}^{10} \left(4z^2 + 4z + 1\right) \varepsilon$

$S = \sum_{z=1}^{10} \left[4z^3 + 4z^2 + 2\right]$

$S = \frac{4 \times 10^2 \times 11^2}{4} + \frac{4 \times 10 \times 11x}{6} + \frac{10 \times 11}{2}$

$S = 13,695$

Question13

The sum of first four terms of a G.P. is 160 and the common ratio is 3 , then the 4^{th} term is MHT CET 2020 (19 Oct Shift 1)

Options:

A. 118

B. 100

C. 108

D. 102

Answer: C

Solution:

a, ar, ar^2, ar^3

$2 = 3$

$\frac{a(\varepsilon^4-1)}{(\varepsilon-1)} = 160$

Solving these we get $a = 4$

$a\varepsilon^3 = t_4 = 108$

Question14

The rational form of a number $1 \cdot \overline{41}$ is MHT CET 2020 (16 Oct Shift 2)

Options:

A. $\frac{154}{99}$

B. $\frac{55}{99}$

C. $\frac{140}{99}$

D. $\frac{41}{99}$

Answer: C

Solution:

The rational form of a number 1.41 is $\frac{140}{99}$.

Let $x = 1.414141$ ...(i) $100x = 141.414141$ ...(ii)

Subtract Eq. (i) from Eq. (ii), we get

$99x = 140$

$x = \frac{140}{99} \therefore 1.\overline{41} = \frac{140}{99}$

Question15

If for an Arithmetic progression, 9 times nineth term is equal to 13 times thirteenth term, then value of twenty second term is MHT CET 2020 (16 Oct Shift 1)

Options:

A. 0

B. 2

C. 4

D. 5

Answer: A

Solution:

Let the first term and common difference of given AP be a and d,

$$9 . a_9 = 13 . a_{13}$$
$$\Rightarrow 9[a + (9 - 1)d] = 13[a + (13 - 1)d]$$
$$\Rightarrow 9[a + 8d] = 13[a + 12d]$$
$$\Rightarrow 9a + 72d = 13a + 156d$$

$$\begin{aligned}\text{respectively.Given} &\Rightarrow 4a + 84d = 0\\ &\Rightarrow 4[a + 21d] = 0\\ &\Rightarrow 4[a + (22 - 1)d = 0\\ &\Rightarrow a + (22 - 1)d = 0\\ &\Rightarrow a_{22} = 0\end{aligned}$$

Question16

If $\frac{1}{4}$, a, b, $\frac{1}{19}$ form a H.P. then the values of a and b **are respectively MHT CET 2020 (15 Oct Shift 1)**

Options:

A. $\frac{1}{12}, \frac{1}{15}$

B. $\frac{1}{5}, \frac{1}{7}$

C. $\frac{1}{9}, \frac{1}{14}$

D. $\frac{1}{11}, \frac{1}{17}$

Answer: C

Solution:

$\frac{1}{4}, a, b, \frac{1}{19} \longrightarrow$ H.P.

$4, \frac{1}{a}, \frac{1}{b}, 19 \rightarrow$ AP

$a = \frac{1}{9}, b = \frac{1}{14}$

Question17

$\frac{1^2}{2}+\frac{1^2+2^2}{3}+\frac{1^2+2^2+3^2}{4}+\frac{1^2+2^2+3^2+4^2}{5}+\ldots\ldots\ldots\ldots$ **upto 8 terms = MHT CET 2020 (14 Oct Shift 2)**

Options:

A. 76

B. 74

C. 78

D. 72

Answer: B

Solution:

$\frac{1^2}{2}+\frac{1^2+2^2}{3}+\frac{1^2+2^2+3^2}{4}+$

general term $\sum_{t=1}^{8}\frac{\sum\varepsilon^2}{\varepsilon}=$

$=\sum_{\varepsilon=1}^{8}\frac{\varepsilon(\varepsilon+1)(2q+1)}{6\varepsilon}$

$=\frac{1}{6}\sum_{\varepsilon=1}^{8}\frac{(2+1)(2\varepsilon+1)}{(8)}$

$=\frac{1}{6}\sum_{\varepsilon=1}^{8}\left[2\varepsilon^2+3r+1\right]$

$=\frac{1}{6}\left[\frac{2\times8\times9\times17}{6}+\frac{3\times8\times9}{2}+8\right]$

$=74$

Question18

For a sequence (t_n) if $s_n=7\left(3^n-1\right)$, then $t_n=$ MHT CET 2020 (14 Oct Shift 1)

Options:

A. $(7)3^{n-1}$

B. $(14)\ 3^{n+1}$

C. $(14)3^{n-1}$

D. $(7)3^{n+1}$

Answer: C

Solution:

A sequence; t_n;

$$s_n=7\left(3^{n-1}\right)$$
$$s_{n-1}=7\left(3^{n-1}-1\right)$$

$$t_n=s_n-s_{n-1}$$
$$t_n=7\left(3^n-3^{n-1}\right)$$
$$=7\cdot 3^{n-1}\cdot 2$$
$$=14\cdot 3^{n-1}$$

Question19

If for the harmonic progression, $t_7=\frac{1}{10}, t_{12}=\frac{1}{25}$, then $t_{20}=$ MHT CET 2020 (13 Oct Shift 2)

Options:

A. $\frac{1}{48}$

B. 49

C. $\frac{1}{49}$

D. 48

Answer: C

Solution:

First term of an $\mathrm{AP}=10$ and the 12^{th} term $=25$. Considering corresponding AP $a+6d=10$ and $a+11d=25d=3, a=-8$

$\Rightarrow T_{20}=a+19d=8+57=49$

Hence, the 20^{th} term of the corresponding HP is $1/49$.

Question20

For a sequence if $S_n=\frac{5^n-2^n}{2^n}$, then its fourth term is MHT CET 2020 (13 Oct Shift 1)

Options:

A. $\frac{375}{16}$

B. $\frac{375}{8}$

C. $\frac{251}{8}$

D. $\frac{251}{16}$

Answer: A

Solution:

$$s_n=\frac{5^n-2^n}{2^n} ; s_{n-1}=\frac{5^{n-1}-2^{n-1}}{2^{n-1}} \quad \begin{aligned} T_n &= s_n-s_{n-1} \\ T_n &= \frac{5^n-2^n \cdot 5^{n-1}}{2^n} \\ T_4 &= \frac{5^4-2 \cdot 5^3}{2^4} \\ &= \frac{625-250}{16} \\ &= \frac{375}{16} \end{aligned}$$

Question21

$$5^2+6^2+7^2+\ldots\ldots\ldots\ldots\ldots+20^2=$$

MHT CET 2020 (12 Oct Shift 2)

Options:

A. 2860

B. 2840

C. 2830

D. 2850

Answer: B

Solution:

The given series is $5^2+6^2+7^2+\ldots+20^2 n^{\text{th}}$ term,

$$a_n=(n+4)^2=n^2+8n+16$$

$$\therefore S_n=\sum_{k=1}^{n} a_k=\sum_{k=1}^{n}\left(k^2+8k+16\right)$$

$$=\sum_{k=1}^{n} k^2+8\sum_{k=1}^{n} k+\sum_{k=1}^{n} 16$$

16^{th} term is

$$=\frac{n(n+1)(2n+1)}{6}+\frac{8n(n+1)}{2}+16n$$

$$(16+4)^2=20^2$$

$$\therefore S_{10}=\frac{16(16+1)(2\times 16+1)}{6}+\frac{8\times 16\times(16+1)}{2}+16\times 16$$

$$=\frac{(16)(17)(33)}{6}+\frac{(8)\times 16\times(16+1)}{2}+16\times 16$$

$$=\frac{(16)(17)(33)}{6}+\frac{(8)(16)(17)}{2}+256$$

$$=1496+1088+256$$

$$=2840$$

$$\therefore 5^2+6^2+7^2+\ldots\ldots+20^2=2840$$

Question22

If $\frac{2+4+6+8-------\text{ upto } n \text{ terms}}{1+3+5+7-------\text{ upto } n \text{ terms}} = \frac{37}{36}$, then $n =$ MHT CET 2020 (12 Oct Shift 1)

Options:

A. 36

B. 29

C. 23

D. 37

Answer: A

Solution:

sum of n even natural numbers $= n(n+1)$

sum of n odd natural numbers $= n\text{^}2$

$n(n+1)/n\text{^}2 = 37/36$

$n + 1/n = 37/36$

cross multiplication

$36(n+1) = 37n$

$36n + 36 = 37n$

$37n - 36n = 36$

$n = 36$

Question23

For a G.P., if $S_n = \frac{4^n - 3^n}{3^n}$, then, $t_2 =$ ________ MHT CET 2019 (02 May Shift 1)

Options:

A. $\frac{1}{9}$

B. $\frac{2}{9}$

C. $\frac{7}{9}$

D. $\frac{4}{9}$

Answer: D

Solution:

$$\text{Given } S_n = \frac{4^n - 3^n}{3^n}$$
$$T_n = S_n - S_{n-1}$$
$$T_2 = S_2 - S_1 = \frac{4^2 - 3^2}{3^2} - \frac{4 - 3}{3}$$
$$= \frac{7}{9} - \frac{1}{3} \Rightarrow \frac{4}{9}$$

Question24

For a G.P., if $m+n^{\text{th}}$ term is p and $m-n^{\text{th}}$ term is $q,$ then m^{th} term is ________ MHT CET 2019 (02 May Shift 1)

Options:

A. pq

B. $\sqrt{pq}$

C. $\frac{p}{q}$

D. $\frac{q}{p}$

Answer: B

Solution:

In a G.P., each term is G. M. of terms which we are equal distance from it

Then, $T_m{}^2 = T_{m+n} \quad T_{m-n}$

then, $\quad T_m = \sqrt{pq}$

Question25

If $\sum_{r=1}^{n}(2r+1)=440$, then n =.....
MHT CET 2019 (Shift 2)

Options:

A. 20

B. 22

C. 21

D. 19

Answer: A

Solution:

We have, $\sum_{r=1}^{n}(2r+l)=440$
$\Rightarrow 3+5+7\ldots+(2n+1)=440$
$\Rightarrow \frac{n}{2}[2\times 3+(n-1)(2)]=440$
$\Rightarrow n(3+n-1)=440$
$\Rightarrow n(n+2)=440$
$\Rightarrow n=20$

Question26

If the sum of an infinite GP be 9 and sum of first two terms be 5 then their common ratio is ….. MHT CET 2019 (Shift 2)

Options:

A. $\frac{1}{3}$

B. 3

C. $\frac{2}{3}$

D. $\frac{3}{2}$

Answer: C

Solution:

We have,
Sum of infinite GP, $\frac{a}{1-r}=9$
$\Rightarrow a+ar=5$
$\Rightarrow a(1+r)=5$
$\Rightarrow 9(1-r)(1+r)=5$
$\Rightarrow 1-r^2=\frac{5}{9}\Rightarrow r^2=\frac{4}{9}\Rightarrow r=\frac{2}{3}$

Question27

For a sequence (t_n), if $S_n=5(2^n-1)$ then $t_n=\ldots$ **MHT CET 2019 (Shift 1)**

Options:

A.

$5(2^{n+1})$

B.

$\frac{5\times 2^n}{4}$

C.

$5(2^{n-1})$

D.

$\frac{2\times(2^{n-1})}{5}$

Answer: C

Solution:

We have, $S_n=5(2^n-1)$
We know that $a_n=S_n-S_{n-1}$
$=5(2^n-1)-5(2^{n-1}-1)$
$=5(2^n-2^{n-1})$
$=5(2^{n-1})$

Question28

If A, B, C are p^{th}, q^{th} and r^{th} terms of a GP respectively then $A^{q-r}.B^{r-p}.C^{p-q}=$ …. MHT CET 2019 (Shift 1)

Options:

A. 0

B. 1

C. 3

D. – 1

Answer: B

Solution:

Let first term and common ratio of a GP are m and n respectively. Then, $A = m\left(n^{p-1}\right) \quad \ldots$ (i)
$B = m\left(n^{q-1}\right) \quad \ldots$ (ii)
and $C = m\left(n^{r-1}\right) \quad \ldots$ (iii)
Now, $A^{q-r} . B^{r-p} . C^{p-q}$
$= m^{(q-r)} . n^{(q-r)(p-1)} . m^{(r-p)} . n^{(r-p)(q-1)} . m^{(p-q)} . n^{(p-q)(r-1)}$
$= m^{q-r+r-p+p-q} . n^{qp-q-rp+r+rq-r-r-pq+p+pr-p-qr+q}$
$= m^0 . n^0 = 1 \times 1 = 1$

Question29

The sum of the first 10 terms of the series $9 + 99 + 999 + . \ldots .$ is MHT CET 2018

Options:

A.

$\frac{9}{8}\left(9^{10} - 1\right)$

B.

$\frac{100}{9}\left(10^9 - 1\right)$

C. 10^9 - 1

D.

$\frac{100}{9}\left(10^{10} - 1\right)$

Answer: B

Solution:

$$S_n = (10 - 1 + 100 - 1 + 1000 - 1 + ...)$$
$$= (10 + 100 + 1000 + ...) - (1 + 1 + 1......)$$
$$= 10\left(\frac{10^n - 1}{10 - 1}\right) - n$$
$$\Rightarrow S_{10} = 10\left(\frac{10^{10} - 1}{9}\right) - 10$$
$$= 10\left(\frac{10^{10} - 1}{9} - 1\right)$$
$$= 10\left(\frac{10^{10} - 1 - 9}{9}\right)$$
$$= 10\left(\frac{10^{10} - 10}{9}\right)$$
$$= \frac{100}{9}\left(10^9 - 1\right)$$
